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AGGREGATION OF CONDITIONAL ABSORBING MARKOV CHAINS.(U)
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Foreword

This paper was presented to the Sixth European Meeting on Cybernetics and Systems Research, which was held at the University of Vienna, April 13-16, 1982.

AGGREGATION OF CONDITIONAL ABSORBING MARKOV CHAINS

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When modeling a process by means of a finite Markov chain, it is sometimes necessary or desirable to stratify the process into subprocesses and model each of these subprocesses. The resulting Markov chain for each subprocess becomes a conditional Markov chain in that its transition probabilities are relative to its associated subprocess. This paper derives the method for aggregating conditional absorbing Markov chains (each of which has the same state space) into a single (unconditional) chain that is representative of the total process and has the same state space as the conditional chains.

1. INTRODUCTION

When modeling a process by means of a finite Markov chain, it is sometimes necessary or desirable to stratify the process into subprocesses and model each of these individual subprocesses. For example, in a study of a distributed data base system [1], the flow of data was modeled as a Markov chain for several separate geographic locations. In a recruiting study [2], the movement of military-age men through the recruiting process and into the armed forces was modeled for separate racial and educational groups as a Markov chain with a single state space for each group—only the input data (transition probabilities) to the model were changed for each group. In [3], a Markov chain model was used to investigate the consequences of induced abortion for different groups of women by estimating transition probabilities separately for each group.

When the above procedure is used, the resulting Markov chain for each subprocess becomes a <u>conditional Markov chain</u> in that its transition probabilities are relative to its associated subprocess. This paper derives the method for aggregating these separate conditional chains (each of which has the same state space) into a single (unconditional) chain that is representative of the total process and has the same state space as the conditional chains.

2. ILLUSTRATIVE EXAMPLE

Figure 1 illustrates a simple four-state process Ψ that has been stratified into two subprocesses Ψ_1 and Ψ_2. Suppose data have been collected for each subprocess (which might represent different geographic regions or different groups of people, for example) and transition probabilities have been estimated as shown in the matrices of transition probabilities P_1 and P_2. Further,

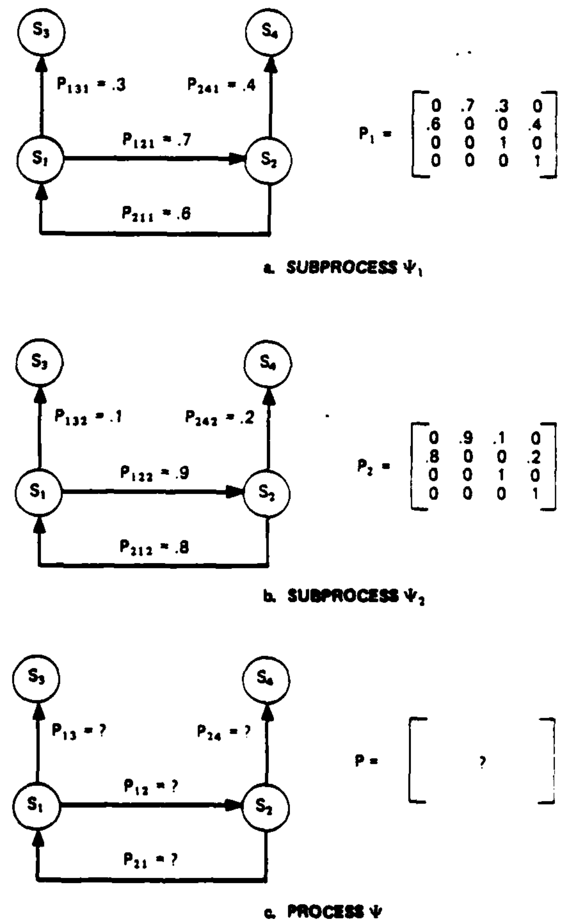

FIG. 1: FOUR-STATE PROCESS WITH TWO SUBPROCESSES

suppose we know the fraction of time (i.e., the probability) that the process originates in each subprocess, say f_1 and f_2, and also know that the process always begins in state S_1. Given this information, how do we determine P? This example illustrates the general problem addressed in this paper. As we shall see later in the paper, what might be considered as two "obvious" methods of determining P do not, in general, work: (1) aggregating the data from the subprocesses and (2) defining $P = f_1P_1 + f_2P_2$.

3. DERIVATION OF P

Suppose the process Ψ under study can be stratified into m subprocesses Ψ_k, $(k = 1, \ldots, m)$, each with the same state space. We assume that Ψ and all the Ψ_k are being modeled as an irreducible, finite, absorbing Markov chain having q transient states and r absorbing states.

If we number the states of the chain so that the transient states "precede" the absorbing states, then the matrix P_k of transition probabilities for subprocess Ψ_k can be partitioned as follows:

$$P_k = (p_{ijk}) = \left[\begin{array}{c|c} Q_k & R_k \\ \hline 0 & I \end{array}\right]$$

where I is the identity matrix of order r, 0 is the r x q zero matrix, R_k is the q x r matrix containing the transition probabilities from transient to absorbing states, and Q_k is the matrix of order q containing the transition probabilities among the transient states.

The probability that Ψ_k is absorbed in state S_j $(j = 1, \ldots, r)$, given that the process began in transient state S_i $(i = 1, \ldots, q)$, is [4]

$$B_k = (b_{ijk}) = (I - Q_k)^{-1}R_k = N_kR_k$$

where N_k is the matrix that gives the expected number N_{ijk} of times that Ψ_k is in each transient state S_j given that it began in each transient state S_i.

We assume, without loss of generality, that each Ψ_k always begins in a particular transient state, say S_1. Then the probability that Ψ_k terminates in S_j is

$$b_{1k} = e_1'B_k = e_1'N_kR_k$$

where e_1 is the unit column vector $e_1 = (\delta_{i1})$, δ_{i1} is the Kronecker delta, and e_1' is the transpose of e_1.

If we let f_k be the probability that Ψ originates in Ψ_k $(\Sigma_k f_k = 1)$, then the probability that the process terminates in S_j is

$$b_1 = \Sigma_k f_k b_{1k} = e_1'\Sigma_k f_k(I - Q_k)^{-1}R_k.$$

Our criterion for determining P is that the limiting probabilities of absorption obtained from P must be the same as those obtained from P_k and f_k. In other words, we want to determine the stochastic matrix P of order q + r such that

$$P = (p_{ij}) = \left[\begin{array}{c|c} Q & R \\ \hline 0 & I \end{array}\right]$$

and $e_1'(I - Q)^{-1}R = e_1'\Sigma_k f_k(I - Q_k)^{-1}R_k$.

To obtain an expression for P, let

N_{1k}^d = a diagonal matrix of order q whose diagonal elements are from the first row of N_k

$$A_k = \left[\begin{array}{c|c} N_{1k}^d & 0 \\ \hline 0 & I \end{array}\right] = \text{a diagonal matrix of order } q + r.$$

Then, as we shall prove in subsequent theorems, the matrix P that satisfies our criterion is

$$P = (\Sigma_k f_k A_k)^{-1}\Sigma_k f_k A_k P_k \quad . \tag{1}$$

Also, if P is given by (1), then the

submatrices Q and R are

$$Q = (\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d Q_k$$

and $R = (\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d R_k$.

4. VERIFICATION OF P

To show that P is in fact the desired matrix we need to show that:

(1) P is stochastic; i.e., $p_{ij} \geq 0$, all i and j, and $\Sigma_j p_{ij} = 1$, all i.

(2) $e_1'(I - Q)^{-1}R = e_1'\Sigma_k f_k (I - Q_k)^{-1} R_k$.

We show that these conditions are met in the following two theorems.

__Theorem 1__. P is stochastic.

__Proof__. From equation (1) it follows that $p_{ij} \geq 0$, all i and j, since each term is nonnegative.

To show that $\Sigma_j p_{ij} = 1$, all i, we need to show that $Pe = e$, where e is the q + r column vector all of whose elements are unity.

Writing P and e in partitioned form,

$$Pe = \left[\begin{array}{c|c} Q & R \\ \hline 0 & I \end{array}\right]\left[\begin{array}{c} e_q \\ \hline e_r \end{array}\right] = \left[\begin{array}{c} Qe_q + Re_r \\ \hline e_r \end{array}\right].$$

Now

$$Qe_q + Re_r = (\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d (Q_k e_q + R_k e_r)$$

$$= (\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d e_q = e_q.$$

Hence $Pe = \left[\begin{array}{c} e_q \\ \hline e_r \end{array}\right] = e$.

Before proving the next theorem we shall need the following

__Lemma__. $e_1'\left[\Sigma_k f_k N_{1k}^d N_k^{-1}\right]^{-1} = e'$.

__Proof__. $e' \Sigma_k f_k N_{1k}^d N_k^{-1} = \Sigma_k f_k (e' N_{1k}^d) N_k^{-1}$

$$= \Sigma_k f_k N_{1k} N_k^{-1}$$

$$= \Sigma_k f_k e_1' = e_1'.$$

Hence $e' = e_1'\left[\Sigma_k f_k N_{1k}^d N_k^{-1}\right]^{-1}$.

The existence of the inverse will be shown in the next theorem.

__Theorem 2__. $e_1'(I - Q)^{-1}R = e_1'\Sigma_k f_k (I - Q_k)^{-1} R_k$.

__Proof__. Consider the first two factors of the left hand side:

$$e_1'(I - Q)^{-1} = e_1'\left[I - (\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d Q_k\right]^{-1}$$

$$= e_1'\left[\Sigma_k f_k N_{1k}^d - \Sigma_k f_k N_{1k}^d Q_k\right]^{-1} \Sigma_k f_k N_{1k}^d$$

$$= e_1'\left[\Sigma_k f_k N_{1k}^d (I - Q_k)\right]^{-1} \Sigma_k f_k N_{1k}^d$$

$$= e_1'\left[\Sigma_k f_k N_{1k}^d (N_k^{-1})\right]^{-1} \Sigma_k f_k N_{1k}^d$$

$$= e' \Sigma_k f_k N_{1k}^d \qquad \text{by the Lemma.}$$

Hence the left hand side becomes

$$e_1'(I - Q)^{-1}R = e'(\Sigma_k f_k N_{1k}^d)(\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d R_k$$

$$= \Sigma_k f_k (e' N_{1k}^d) R_k .$$

Now $e' N_{1k}^d = N_{1k}$, the first row of N_k, therefore

$$e_1'(I - Q)^{-1}R = \Sigma_k f_k N_{1k} R_k = \Sigma_k f_k (e_1' N_k) R_k$$

$$= e_1' \Sigma_k f_k N_k R_k = e_1' \Sigma_k f_k (I - Q_k)^{-1} R_k,$$

which completes the proof.

It is now seen that the matrix $\Sigma_k f_k N_{1k}^d N_k^{-1}$ considered in the Lemma is nonsingular since it is the product of two nonsingular matrices:

$$\Sigma_k f_k N_{1k}^d N_k^{-1} =$$

$$\Sigma_k f_k N_{1k}^d \left[I - (\Sigma_k f_k N_{1k}^d)^{-1} \Sigma_k f_k N_{1k}^d Q_k\right].$$

5. EXPECTED TIME TO ABSORPTION

Another quantity of interest is the expected time to absorption in a given absorbing state

S_j, given that Ψ_k began in transient state S_i. The matrix of transition probabilities for Ψ_k conditioned on the hypothesis that Ψ_k is absorbed in S_j is [4]

$$P_{jk} = \left[\begin{array}{c|c} Q_{jk} & e'(I - Q_{jk}) \\ \hline 0 & 1 \end{array}\right]$$

where $Q_{jk} = D_{jk}^{-1} Q_k D_{jk}$

and $D_{jk} = \begin{bmatrix} b_{1jk} & & 0 \\ & \ddots & \\ 0 & & b_{qjk} \end{bmatrix}$

is a diagonal matrix of order q formed from column j of B_k. P_{jk} is of order q + 1, $e'(I - Q_{jk})$ is a q-component column vector, and 0 is the q-component zero row vector.

Given that Ψ_k is absorbed in state S_j, the expected time to absorption is [1]

$$V_{jk} = (I - Q_{jk})^{-1} T_k = N_{jk} T_k$$

where $T_k = (\bar{t}_{ik})$ is a column vector whose elements are the expected times $\bar{t}_{ik}$ that Ψ_k spends in each transient state S_i.

Since Ψ_k always begins in transient state S_1, the expected time for Ψ_k to be absorbed in state S_j is

$$e_1' V_{jk} = e_1'(I - Q_{jk})^{-1} T_k$$

and the expected time for the overall process Ψ to be absorbed in state S_j is

$$e_1' V_j = \Sigma_k f_k e_1' V_{jk} = e_1' \Sigma_k f_k (I - Q_{jk})^{-1} T_k .$$

To determine the matrix P_j and the time vector T for Ψ under the hypothesis that Ψ is absorbed in S_j, we use the criterion that the expected time to absorption obtained by using P_j and T must be the same as the time obtained by using P_{jk}, T_k, and f_k. In other words, we want to determine P_j and T such that

$$P_j = \left[\begin{array}{c|c} Q_j & e'(I - Q_j) \\ \hline 0 & 1 \end{array}\right]$$

and $e_1'(I - Q_j)^{-1} T = e_1' \Sigma_k f_k (I - Q_{jk})^{-1} T_k$.

To obtain expressions for P_j and T, let

N^d_{1jk} = the diagonal matrix of order q whose diagonal elements are from the *first row* of N_{jk},

and $A_{jk} = \left[\begin{array}{c|c} N_{1jk} & 0 \\ \hline 0 & 1 \end{array}\right]$.

Then

$$P_j = (\Sigma_k f_k A_{jk})^{-1} \Sigma_k f_k A_{jk} P_{jk} ,$$

$$Q_j = (\Sigma_k f_k N^d_{1jk})^{-1} \Sigma_k f_k N^d_{1jk} Q_{jk} ,$$

and $$T = (\Sigma_k f_k N^d_{1jk})^{-1} \Sigma_k f_k N^d_{1jk} T_k .$$

The proofs that P_j is stochastic and that

$$e_1'(I - Q_j)^{-1} T = e_1' \Sigma_k f_k (I - Q_{jk})^{-1} T_k$$

are the same as those given in Theorems 1 and 2.

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